We study quantum phase transitions in the half-filled Hubbard model on the triangular lattice by means of the path-integral renormalization group method (PIRG) [1] with an iteration scheme proposed recently [2]. It is found in the system with 36 sites that the Hubbard interaction $U$ induces the first-order phase transition from the paramagnetic metallic state to a nonmagnetic insulating state at $U_{c1} \sim 7.4t$, which is followed by another first-order transition to a 120 degrees Neel ordered state at $U_{c2} \sim 9.2t$, where $t$ is the transfer integral. Our reliable results obtained from finite clusters with 16, 24, 30, and 36 sites suggest the existence of the intermediate nonmagnetic insulating state in the thermodynamic limit [3] and resolve some controversial arguments on the nature of the previously proposed quantum phase transitions [4].